División en forma polar

10/06/2011

Si $Z_1 = r_1 \; \alpha$ y $Z_2 = r_2 \; \beta$ → Ángulo de Z_2

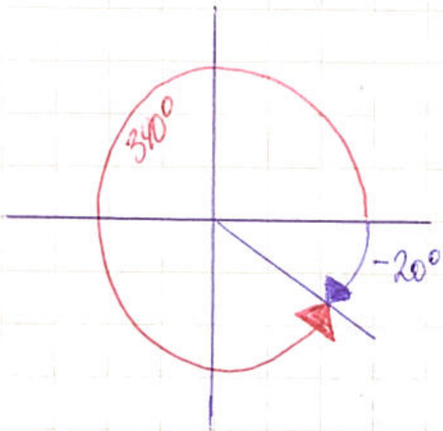
módulo de Z_1 — módulo de Z_2

$$\frac{Z_1}{Z_2} = \left(\frac{r_1}{r_2}\right)(\alpha - \beta)$$

Ejemplo: $Z_1 = 5_{30°}$ $Z_2 = 2_{50°}$

a) $\frac{Z_1}{Z_2} = \left(\frac{5}{2}\right)_{(30° - 50°)} = \left(\frac{5}{2}\right)_{(-20°)} = \left(\frac{5}{2}\right)_{(340°)}$ //

b) $\frac{Z_2}{Z_1} = \left(\frac{2}{5}\right)_{(50° - 30°)} = \left(\frac{2}{5}\right)_{(20°)}$



Potencia de un complejo en forma Polar

En forma canónica:

$$(1+\sqrt{3}i)^2 = 1^2 + 2\cdot 1\cdot \sqrt{3}i + (\sqrt{3}i)^2$$
$$= 1 + 2\sqrt{3}i + \sqrt{3}^2 \cdot i^2$$
$$= 1 + 2\sqrt{3}i + 3\cdot -1$$
$$= 1 + 2\sqrt{3}i - 3$$
$$= 2\sqrt{3}i - 2$$

En forma polar:

$$(r_\alpha)^2 = r_\alpha \cdot r_\alpha = (r\cdot r)_{(\alpha+\alpha)} = r^2{}_{2\alpha}$$
$$(r_\alpha)^3 = r_\alpha \cdot r_\alpha \cdot r_\alpha = (r\cdot r\cdot r)_{(\alpha+\alpha+\alpha)} = r^3{}_{3\alpha}$$
$$(r_\alpha)^4 = r_\alpha \cdot r_\alpha \cdot r_\alpha \cdot r_\alpha = (r\cdot r\cdot r\cdot r)_{(\alpha+\alpha+\alpha+\alpha)} = r^4{}_{4\alpha}$$
$$\vdots$$
$$(r_\alpha)^n = r^n{}_{n\alpha} \;//$$

Recordemos que:

$$\underbrace{1+\sqrt{3}i}_{\text{Canónica}} = \underbrace{2_{60°}}_{\text{Polar}}$$

Por lo tanto:

$$(1+\sqrt{3}i)^2 = (2_{60°})^2 = $$
$$2^2{}_{2\cdot 60°} = 4_{120°} \;//$$

En resúmen, cada operación en forma polar sería:

$r_{1\,\alpha} \cdot r_{2\,\beta}$

Si, $a + bi = r_1 \cdot \cos\alpha + r_2 \cdot \text{sen}\,\alpha\, i = r_{1\,\alpha}$

$c + di = r_2 \cdot \cos\beta + r_2 \cdot \text{sen}\,\beta i = r_{2\,\beta}$

$$\boxed{\begin{aligned}(a+bi)(c+di) &= r_{1\,\alpha} \cdot r_{2\,\beta} = (r_1 \cdot r_2)_{(\alpha+\beta)} = \\ &r_1 \cdot r_2 \cos(\alpha+\beta) + \sqrt{1} \cdot \sqrt{2} \cdot \text{sen}(\alpha+\beta) i\end{aligned}}$$

$$\frac{a+bi}{c+di} = \frac{r_{1\,\alpha}}{r_{2\,\beta}} = \left(\frac{r1}{r2}\right)_{(\alpha-\beta)} =$$

$$\left(\frac{r1}{r2}\right)\cos(\alpha-\beta) + \left(\frac{r1}{r2}\right)\text{sen}\,(\alpha-\beta i)$$

Ejemplos:

Consideramos:

$Z1 = 2 + \sqrt{2}\, i$ y $Z_2 = 2 - \sqrt{2}\, i$

Resolver: $Z_1 \cdot Z_2$ y $\frac{Z1}{Z_2}$ en 3 formas:

En forma canónica:

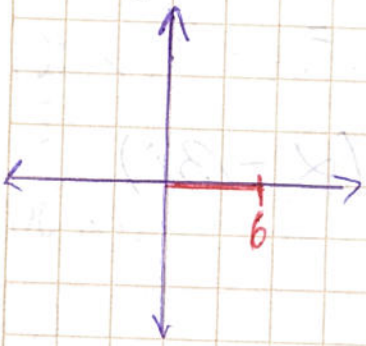
$$Z_1 \cdot Z_2 = (2+\sqrt{2}i)(2-\sqrt{2}i) = 2^2 - (\sqrt{2}i)^2$$
$$= 4 - 2 \cdot -1$$
$$= 4 + 2$$
$$= 6$$

$$|Z_1 \cdot Z_2| = 6$$

$$\alpha = 0°$$

Luego, en forma trigonométrica:

$$Z_1 \cdot Z_2 = 6 \cos 0° + 6 \operatorname{sen} 0° i$$

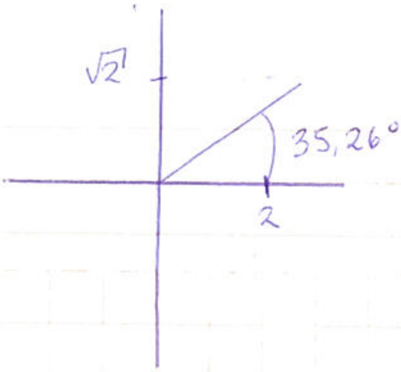


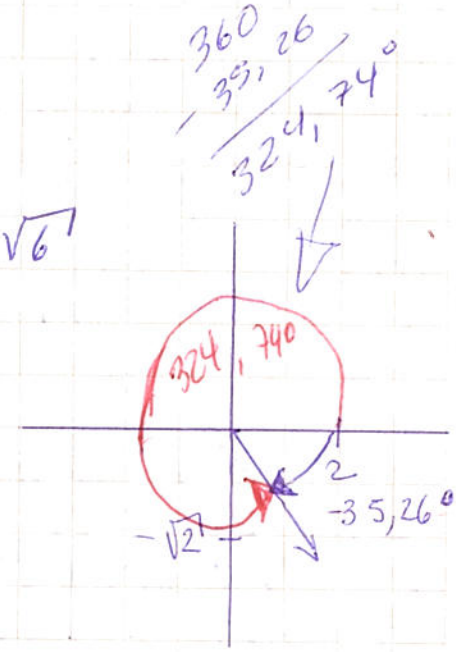
En forma polar: Debemos calcular el módulo y el ángulo de cada complejo (Z_1 y Z_2)

$$|Z_1| = |2+\sqrt{2}i| = \sqrt{2^2 + \sqrt{2}^2}$$
$$= \sqrt{4+2}$$
$$= \sqrt{6}$$

Ángulo de Z_1 =

$$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) = 35,26°$$





Transformaremos Z_2

$|Z_2| = \sqrt{2^2+(-\sqrt{2})^2} = \sqrt{4+2} = \sqrt{6}$

Ángulo de $Z_2 =$

$\tan^{-1}\left(\frac{-\sqrt{2}}{2}\right) = -35,26°$

$Z_2 = 2 - \sqrt{2}\,i$

Luego, Z_2 en forma polar es: $\sqrt{6}\ 324,74°$

Si multiplicamos $Z1 \cdot Z2$ en forma polar, queda:

$Z1 \cdot Z2 = \sqrt{6}\ 35,26° \cdot \sqrt{6}\ 324,74° =$

$\sqrt{6} \cdot \sqrt{6}\ (35,26° + 324,74°) = 6\ 360° = 6\ 0°$

En forma trigonométrica:

Cómo $|Z1| = \sqrt{6}$ y el ángulo de $Z_1 = 35,26°$

$Z1$ en forma trigonométrica es:

$\sqrt{6} \cdot \cos 35,26° + \sqrt{6} \cdot \operatorname{sen} 35,26°$

Del mismo:

$|Z_2| = \sqrt{6}$ y el ángulo es 324, 74°, luego

Z_2 en forma trigonométrica es:

$\sqrt{6} \cdot \cos 324{,}74° + \sqrt{6} \operatorname{sen} 324{,}74° i$

luego $Z1 \cdot Z_2$ en forma trigonométrica sería:

$$\boxed{\begin{array}{l}\sqrt{6} \cdot \sqrt{6} \cos(35{,}26° + 324{,}74°) + \\ \sqrt{6} \cdot \sqrt{6} \operatorname{sen}(35{,}26° + 324{,}74°) i\end{array}}$$

$\Downarrow$ $\Downarrow$

$6 \underbrace{\cos(360°)}_{1} + 6 \underbrace{\operatorname{sen}(360°)}_{0} i = 6$ //

$6 \cdot 1 + 6 \cdot 0 \cdot i = 6 + 0 = 6$ //

División:

En forma canónica:

$$\frac{(2+\sqrt{2}\,i)}{(2-\sqrt{2}\,i)} \cdot \frac{(2+\sqrt{2}\,i)}{(2+\sqrt{2}\,i)} = \frac{2^2+2\cdot 2\sqrt{2}\,i+\sqrt{2}^2\cdot i^2}{2^2-(\sqrt{2}\,i)^2} =$$

$$\frac{4+4\sqrt{2}\,i-2}{6} = \frac{2+4\sqrt{2}\,i}{6} = \frac{2}{6}+\frac{4}{6}\sqrt{2}\,i$$

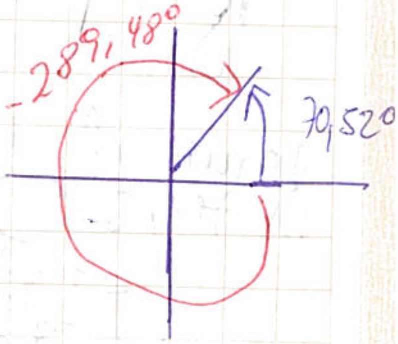
$$= \frac{1}{3}+\frac{2}{3}\sqrt{2}\,i$$

Próximas Evaluaciones

- 15 julio → 2º Solemne
- 29 julio → Exámen → Todo la materia

Temario 2do Solemne
- Forma trigonométrica de un complejo
- Vectores.

En forma polar:



$$Z1 = 2+\sqrt{2}\,i = \sqrt{6}\ \ 35{,}26°$$

$$Z2 = 2-\sqrt{2}\,i = \sqrt{6}\ \ 324{,}74°$$

$$\frac{Z1}{Z2} = \frac{\sqrt{6}\ \ 35{,}26°}{\sqrt{6}\ \ 324{,}74°} = \left(\frac{\sqrt{6}}{\sqrt{6}}\right)(35{,}26° - 324{,}74°) =$$

$$1\,(-289{,}48°) = 1\ \ 70{,}52°$$

Si transformamos el resultado en forma polar, es:

$$Z1 \cdot Z2 = \frac{1}{3} + \frac{2}{3}\sqrt{2}\,i$$

$$|Z1 \cdot Z_2| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\sqrt{2}\right)^2}$$

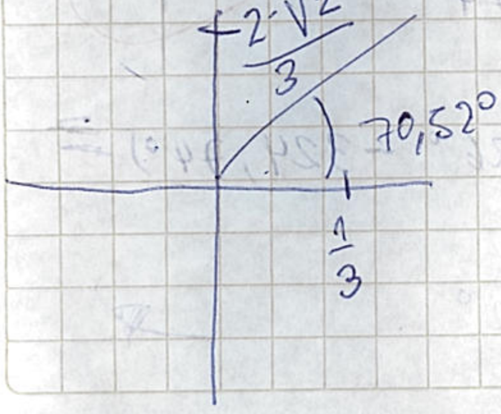
$$= \sqrt{\frac{1}{9} + \frac{4 \cdot 2}{9}} = \sqrt{\frac{1}{9} + \frac{8}{9}} = \sqrt{\frac{9}{9}} =$$

$$1 \,//$$

El ángulo de $Z1 : Z_2$ en forma canónica sería $tg^{-1}\left(\frac{\frac{2 \cdot \sqrt{3}}{3}}{\frac{1}{3}}\right) = tg^{-1}\left(\frac{2 \cdot \sqrt{2}}{\cancel{3}} \cdot \frac{3}{1}\right)$

$$= tg^{-1}\left(2 \cdot \sqrt{2}\right) = 70,52^\circ$$

Como el complejo es del 1er cuadrante, el ángulo es $70,52^\circ$



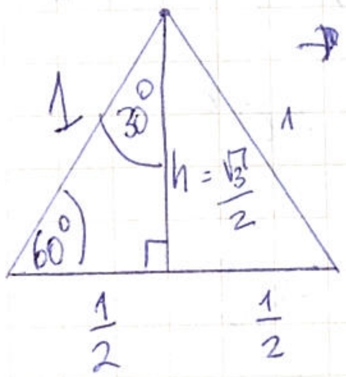
En forma trigonométrica, $Z1 : Z2$ es igual a raíz de $\frac{\sqrt{6}}{\sqrt{6}} \Rightarrow 1 \cdot$ coseno$(35,26° - 324,74°) + \frac{\sqrt{6}}{\sqrt{6}} 1 \cdot$ sen$(35,26° - 324,74°)$

$= \cos(-289,48) + \operatorname{sen}(-289,48)\, i$

$= \cos(70,52°) + \operatorname{sen}(70,52°)\, i$

Ojo con esta tabla: Ángulos notables

	30°	45°	60°
seno	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
coseno	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tangente	$\sqrt{3}/3$	1	$\sqrt{3}$



→ triángulo Equilátero

$1^2 = \left(\frac{1}{2}\right)^2 + h^2$

$1 = \frac{1}{4} + h^2$

$1 - \frac{1}{4} = h^2$

$3/4 = h^2 \ /\sqrt{\ }$

$\sqrt{3/4} = h \rightarrow h = \frac{\sqrt{3}}{\sqrt{4}} \rightarrow \boxed{h = \frac{\sqrt{3}}{2}}$

Cómo se calculan

$$\text{sen } 30^\circ = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\text{tg } 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 30^\circ$$

$6 + 2\sqrt{3}i$ El ángulo de ese complejo sería

$$\tan^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{6}\,3}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30^\circ$$

¿Qué ángulo tiene $\text{tg}^{-1} = \frac{\sqrt{3}}{3} \Longrightarrow 30^\circ$