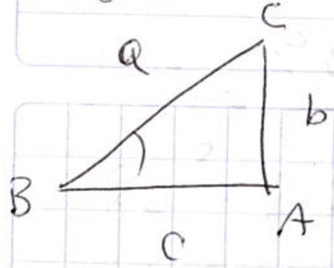
Razones trigonométricas 03/6/2011



Seno (B)

Coseno (B)

tangente (B)

c b = Catetos

a = hipotenusa

c = Cateto contiguo o adyacente a B

b = Cateto opuesto a B

$$\text{Seno }(B) = \frac{\text{Cateto opuesto a } B}{\text{Hipotenusa}} = \frac{b}{a}$$

$$\text{Coseno }(B) = \frac{\text{Cateto contiguo a } B}{\text{Hipotenusa}} = \frac{c}{a}$$

$$\text{tangente }(B) = \frac{\text{Cateto Opuesto a } B}{\text{Cateto Contiguo a } B} = \frac{b}{c}$$

Seno $c = \frac{c}{a}$

Teorema:

Si $\alpha + \beta = 90°$

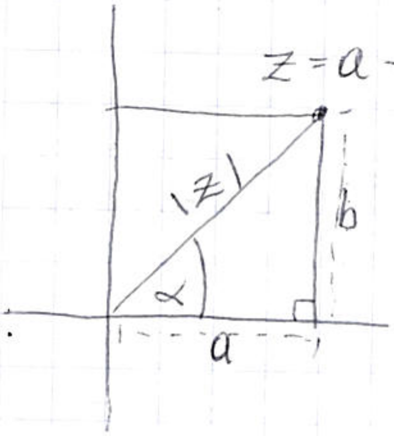
Entonces

Seno α = cos β

cos α = Seno β

Coseno $c = \frac{b}{a}$

tangente $c = \frac{c}{b}$



Seno $\alpha = \frac{b}{|z|} \Rightarrow \boxed{b = |z| \cdot \text{sen}\,\alpha}$

Coseno $\alpha = \frac{a}{|z|} \Rightarrow \boxed{a = |z| \cdot \cos\alpha}$

$$a + bi = \underbrace{\boxed{|z| \cdot \cos\alpha + |z| \cdot \text{sen}\,\alpha \cdot i}}_{\text{forma geométrica de un complejo}}$$

El ángulo α se llama argumento y se calcula utilizando la siguiente ecuación

$$\tan^{-1}(tg(\alpha)) = \alpha \quad \text{(calculadora)}$$

$$\tan^{-1}\left(\frac{b}{a}\right) = \alpha$$

Interpretación del resultado de:

$\tan^{-1}\left(\frac{b}{a}\right) = \alpha$

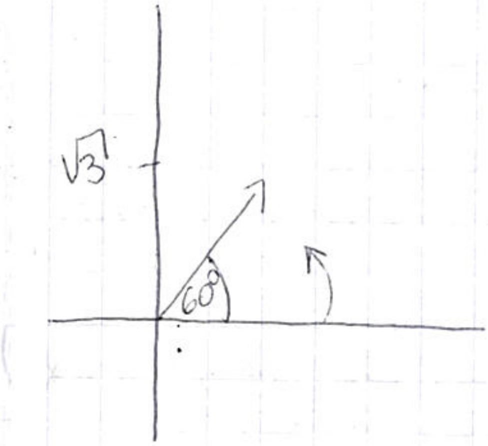
1) $z = 1 + \sqrt{3}\, i$

$|z| = \sqrt{1^2 + \sqrt{3}^2}$

$= \sqrt{1+3}$

$\rightarrow |z| = \sqrt{4} = 2$

$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60^\circ$



$z = 2\cos 60^\circ + 2 \cdot \operatorname{sen} 60^\circ i$

2) $z = -1 + \sqrt{3}\, i = (-1, \sqrt{3})$

$|z| = \sqrt{(-1)^2 + \sqrt{3}^2}$

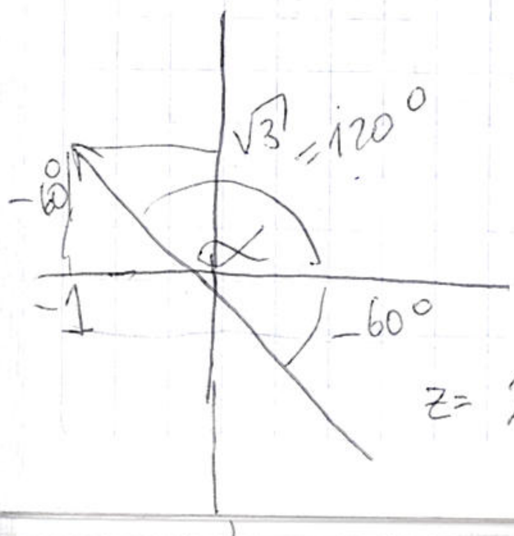
$|z| = \sqrt{1+3}$

$|z| = 2$

$\tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = -60^\circ \Rightarrow \alpha = 120^\circ$

En el 2° cuadrante

$\alpha = \left(180^\circ + \tan^{-1}\left(\frac{b}{a}\right)\right)$

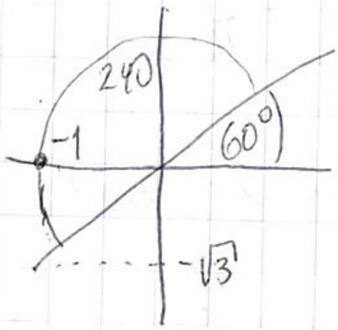


$z = 2 \cdot \cos 120^\circ + 2 \cdot \operatorname{sen} 120^\circ i$

③ $z = -1 - \sqrt{3}\,i$

$|z| = 2$

$$\tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}(\sqrt{3}) = 60°$$



En el 3er cuadrante

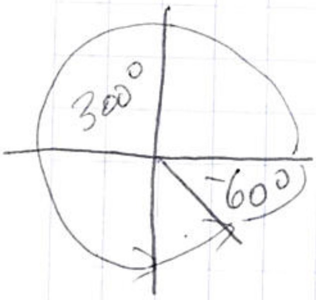
$$\alpha = \left(180° + \tan^{-1}\left(\frac{b}{a}\right)\right)$$

$$z = 2 \cdot \cos 240° + 2 \cdot \operatorname{Sen} 240°\, i$$

$-1 - \sqrt{3}$

④ $z = 1 - \sqrt{3}\,i$ $\quad |z| = 2$

$$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(-\sqrt{3}) = -60°$$



En el cuadrante 4,

$$\alpha = \left(360° + \tan^{-1}\left(\frac{b}{a}\right)\right)$$

$$z = 2 \cdot \cos 300° + 2 \cdot \operatorname{Sen} \cdot 300°\, i$$

Forma Polar:

Con la norma del complejo y el ángulo, se puede resumir la forma trigonométrica a la forma polar, llamamos $r = |z|$,

entonces la forma polar es

$$|z| \cdot \cos \alpha + |z| \cdot \operatorname{sen} \alpha \cdot i = r_\alpha$$

Ej:

$$z = 2 \cdot \cos 300° + 2 \operatorname{sen} 300° i = 2_{300°}$$

Operaciones en forma polar.

1° Multiplicación de complejos

Si $z_1 = r_\alpha$ y $z_2 = r'_\beta$,

$$z_1 \cdot z_2 = r_\alpha \cdot r'_\beta = r \cdot r'_{(\alpha + \beta)}$$

Ej: $z_1 = 1 + \sqrt{3} i$

$z_2 = -1 + \sqrt{3} i$

$$z_1 \cdot z_2 = (1 + \sqrt{3} i)(-1 + \sqrt{3} i)$$

$$= 1 + \sqrt{3} i - \sqrt{3} i + 3 i^2$$

$$= -1 - 3$$

$$z_1 \cdot z_2 = -4$$

$r = 4$

$\alpha = 180°$

En forma polar

$z_1 = 1 + \sqrt{3}\,i = 2_{60°}$

$z_2 = -1 + \sqrt{3}\,i = 2_{120°}$

$z_1 \cdot z_2 = 2_{60°} \cdot 2_{120°} = 2 \cdot 2\,(60° + 120°) = 4_{180°}$

Ejercicios:

Multiplicar en forma polar y comprobar en forma algebraica:

a) $z_1 \cdot z_4$ b) $z_2 \cdot z_3$ c) $z_2 \cdot z_4$

$z_1 = 2_{60°}$ $z_3 = 2_{240°}$

$z_2 = 2_{120°}$ $z_4 = 2_{300°}$

a) $z_1 \cdot z_4 =$

$2_{60°} \cdot 2_{300°} = 2 \cdot 2\,(60° + 300°) = 4\,(360°)$

$= 4\,(0)$

$= 4_0$

comp.

$z_1 = 1 + \sqrt{3}\,i$ $z_4 = 1 - \sqrt{3}\,i$

$(1 + \sqrt{3}\,i)(1 - \sqrt{3}\,i)$

$(1 - \sqrt{3}\,i + \sqrt{3}\,i - \sqrt{3}^2\,i^2)$

$1 - 3 \cdot i^2$ $\rightarrow$ $1 - 3 \cdot -1$

$1 + 3$

2 //

b) $z_2 \cdot z_3$

$$2_{120^\circ} \cdot 2_{240^\circ} = 4(120+240)$$

$$4(360) = 4_0$$

$z_2 = -1+\sqrt{3}i$

$z_3 = -1-\sqrt{3}i$

$$(-1+\sqrt{3}i)\cdot(-1-\sqrt{3}i) =$$

$$1+\sqrt{3}i-\sqrt{3}i-\sqrt{3}^2 i^2$$

$$1-3\cdot -1 = 1+3 = 4$$

c) $z_2 \cdot z_4$

$$2_{120^\circ} \cdot 2_{300^\circ} = 4(120^\circ+300^\circ)$$

$$4(420^\circ) = 4(60)$$

$$(-1+\sqrt{3}i)(1-\sqrt{3}i)$$

$$-1+\sqrt{3}i+\sqrt{3}i-\sqrt{3}^2 i^2$$

$$-1+2\sqrt{3}i-3\cdot -1$$

$$2+2\sqrt{3}i = \sqrt{2^2+2\sqrt{3}^2}$$

$$= \sqrt{4+12} = 4$$

$$tg^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right) = 60^\circ$$